

Tutorial 3

Advanced Graph Theory

August 6, 2013

1. Prove or Disprove:

- 1.1 Every Eulerian bipartite graph has an even number of edges.
- 1.2 Every Eulerian simple graph with an even number of vertices has an even number of edges.
- 1.3 If G is an Eulerian graph, and there are two edges e and f in G sharing a vertex, then G has a eulerian circuit in which e and f appear consecutively.

2. Suppose that G is a graph and D is an orientation of G that is strongly connected. Prove that if G has an odd cycle, then D has an odd cycle.
3. Which of the following are graphic sequences?
 - 3.1 $(5,5,4,3,2,2,2,1)$
 - 3.2 $(5,5,4,4,2,2,1,1)$
 - 3.3 $(5,5,5,3,2,2,1,1)$
 - 3.4 $(5,5,5,4,2,1,1,1)$

4. Suppose a connected graph G is decomposed into two graphs G_1 and G_2 . Prove that G_1 and G_2 must have a common vertex.
5. Given a hypercube Q_r , find the smallest and largest length cycle such that the cycle covers all r dimensions.
6. Create an example of a graph in which every vertex is a king. Can you generalize this into a class of graphs for which every vertex becomes a king?